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Type 0 Brane Inflation from Mirage Cosmology

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Abstract

We consider a three-dimensional brane-universe moving in a Type 0 String background. The motion induces on the brane a cosmological evolution which, for some range of the parameters, exhibits an inflationary phase.

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1 Introduction

The proposal that our observable four-dimensional universe is a domain wall embedded in a higher dimensional space [1], is intensively investigated in various contexts. In an earlier speculation, motivated by the long standing hierarchy problem, it was proposed [2] that the fundamental Planck scale could be close to the gauge unification scale, at the price of "large" spatial dimensions, the introduction of which explains the observed weakness of gravity at long distances. In a similar scenario [3], our observed world is embedded in a five-dimensional bulk, which is strongly curved. This allows the extra dimension not to be very large, and we can perceive gravity as effectively four-dimensional.

This idea of a brane-universe can naturally be applied to string theory. In this context, the Standard Model gauge bosons as well as charged matter arise as fluctuations of the D-branes. The universe is living on a collection of coincident branes, while gravity and other universal interactions is living in the bulk space [4]. For example, the strongly coupled $E_8 \times E_8$ heterotic string theory is believed to be an eleven-dimensional theory, the field theory limit of which was described by Horava and Witten [5]. The spacetime manifold includes a compact dimension with an orbifold structure. Matter is confined on the two ten-dimensional hypersurfaces (nine-branes) which can be seen as forming the boundaries of this spacetime.

For all such theories, an essential issue concerns the cosmological evolution of our universe. In the literature, there are a lot of cosmological models associated to brane universe [6]-[9]. In these models one can recognize two main approaches. In the first one, the domain walls (branes) are static solutions of the underlying theory, and the cosmological evolution of our universe is due to the time evolution of energy density on the domain wall (brane) [7]. In the second approach, the cosmological evolution of our universe is due to the motion of our brane-world in the background gravitational field of the bulk [8, 9].

In [8] the motion of a domain wall (brane) in a higher dimensional spacetime was studied. The Israel matching conditions were used to relate the bulk to the domain wall (brane) metric, and some interesting cosmological solutions were found. In [9] a universe three-brane is considered in motion in ten-dimensional space in the presence of a gravitational field of other branes. It was shown that this motion in ambient space induces cosmological expan-

sion (or contraction) on our universe, simulating various kinds of matter.

In this work we will examine the motion of a three-brane in a background of the type 0 string theory. Type 0 string theories [10] are interesting because of their connection [11] to four-dimensional $SU(N)$ gauge theory. The type 0 string does not have spacetime supersymmetry and because of that it contains in its spectrum a non-vanishing tachyon field. In spite of the presence of the tachyon field these theories in an AdS background are stable [12].

Employing the technics of ref. [9], we will show that a three-brane as it moves in the type 0 string bulk, it inflates. Stability of the theory requires that the tachyon field takes a constant value [10], and taking also a constant value for the dilaton field, the theory can be solved exactly in an $AdS \times S^5$ background. We will show that this metric induces on the brane a cosmological evolution which for some range of the parameters has an inflationary epoch. As we will discuss in the following, the tachyon function $f(T)$ which couples to the F_5 form of the type 0 strings, is crucial for the inflationary evolution of the brane-universe.

Our work is organized as follows. In section two, we briefly review the technics of ref. [9]. In section three, we discuss the type 0 string and for constant dilaton and tachyon fields we give an exact solution in an $AdS \times S^5$ background. In section four, using this solution we find the cosmological evolution of the three-brane in a background of type 0 string. Finally in the last section we discuss our results.

2 Brane-Universe

In [9] it was considered a D-brane moving in a generic static, spherically symmetric background. As the brane moves in a geodesic, the induced world-volume metric becomes a function of time, so there is a cosmological evolution from the brane point of view. The metric of a D3-brane is parametrized as

$$ds_{10}^2 = g_{00}(r)dt^2 + g(r)(d\vec{x})^2 + g_{rr}(r)dr^2 + g_S(r)d\Omega_5 \tag{1}$$

and there is also a dilaton field Φ as well as a RR background $C(r) = C_{0\ldots3}(r)$ with a self-dual field strength. The action on the brane is given by

$$S = T_3 \int d^4\xi e^{-\Phi}\sqrt{-det(\hat{G}_{\alpha\beta} + (2\pi\alpha')F_{\alpha\beta} - B_{\alpha\beta})}$$

$$+T_3 \int d^4\xi \hat{C}_4 + anomaly\ terms \tag{2}$$

The induced metric on the brane is

$$\hat{G}_{\alpha\beta} = G_{\mu\nu} \frac{\partial x^\mu \partial x^\nu}{\partial \xi^\alpha \partial \xi^\beta} \tag{3}$$

with similar expressions for $F_{\alpha\beta}$ and $B_{\alpha\beta}$.In the static gauge, using (3) we can calculate the bosonic part of the brane Lagrangian which reads

$$L = \sqrt{A(r) - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\varphi}^i\dot{\varphi}^j} - C(r) \tag{4}$$

where $h_{ij}d\varphi^i d\varphi^j$ is the line element of the unit five-sphere,and

$$A(r) = g^3(r)|g_{00}(r)|e^{-2\Phi}, B(r) = g^3(r)g_{rr}(r)e^{-2\Phi}, D(r) = g^3(r)g_S(r)e^{-2\Phi} \tag{5}$$

Demanding conservation of energy E and of total angular momentum ℓ^2 on the brane, the induced four-dimensional metric on the brane is

$$d\hat{s}^2 = (g_{00} + g_{rr}\dot{r}^2 + g_S h_{ij}\dot{\varphi}^i\dot{\varphi}^j)dt^2 + g(d\vec{x})^2 \tag{6}$$

with

$$\dot{r}^2 = \frac{A}{B}(1 - \frac{A}{(C+E)^2}\frac{D+\ell^2}{D}), h_{ij}\dot{\varphi}^i\dot{\varphi}^j = \frac{A^2\ell^2}{D^2(C+E)^2} \tag{7}$$

Using (7), the induced metric becomes

$$d\hat{s}^2 = -d\eta^2 + g(r(\eta))(d\vec{x})^2 \tag{8}$$

with η the cosmic time which is defined by

$$d\eta = \frac{|g_{00}|g^{\frac{3}{2}}e^{-\Phi}}{|C+E|}dt \tag{9}$$

This equation is the standard form of a flat expanding universe. If we define the scale factor as $\alpha^2 = g$ then we can calculate the Hubble constant $H = \frac{\dot{\alpha}}{\alpha}$, where dot stands for derivative with respect to cosmic time. Then

we can interpret the quantity $(\frac{\dot{\alpha}}{\alpha})^2$ as an effective matter density on the brane with the result

$$\frac{8\pi}{3}\rho_{eff} = \frac{(C+E)^2 g_S e^{2\Phi} - |g_{00}|(g_S g^3 + \ell^2 e^{2\Phi})}{4|g_{00}|g_{rr}g_S g^3}(\frac{g'}{g})^2 \tag{10}$$

Therefore the motion of a D3-brane on a general spherically symmetric background had induced on the brane a matter density. As it is obvious from the above relation, the specific form of the background will determine the cosmological evolution on the brane.

3 Type 0 string background

Type 0 string theory is interesting because except its connection to gauge theories it contains a tachyon field. Tachyonic fields in ordinary field theory create instabilities. In cosmology on the contrary, the time evolution of a tachyon field plays an important *rôle*. In two dimensions because the tachyon field is a matter field has important consequences in cosmology [13], and it can give a solution to the "gracefull exit" problem [14]. In four dimensions its effect to cosmology has been examined by various authors [15]. We will show that in the type 0 string the tachyon field can induce inflation on the brane. The action of the type 0 string is given by [10]

$$\begin{aligned} S_{10} \;=\; & \int d^{10}x\sqrt{-g}\Big[e^{-\Phi}\Big(R + (\partial_\mu\Phi)^2 - \frac{1}{4}(\partial_\mu T)^2 - \frac{1}{4}m^2T^2 - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho}\Big) \\ & -\frac{1}{2}(1 + T + \frac{T^2}{2})|F_5|^2\Big] \end{aligned} \tag{11}$$

The equations of motion which result from this action are

$$2\nabla^2\Phi - 4(\nabla_n\Phi)^2 - \frac{1}{2}m^2T^2 = 0 \tag{12}$$

$$\begin{aligned} R_{mn} \; & +2\nabla_m\nabla_n\Phi - \frac{1}{4}\nabla_m T\nabla_n T - \frac{1}{4\cdot 4!}e^{2\Phi}f(T)\Big(F_{mklpq}F_n{}^{klpq} \\ & -\frac{1}{10}G_{mn}F_{sklpq}F^{sklpq}\Big) = 0 \end{aligned} \tag{13}$$

$$(-\nabla^2 + 2\nabla^n\Phi\nabla_n + m^2)T + \frac{1}{2\cdot 5!}e^{2\Phi}f'(T)F_{sklpq}F^{sklpq} = 0 \tag{14}$$

$$\nabla_m\Big(f(T)F^{mnkpq}\Big) = 0 \tag{15}$$

The tachyon is coupled to the RR field through the function

$$f(T) = 1 + T + \frac{1}{2}T^2 \tag{16}$$

In the background where the tachyon field acquires vacuum expectation value $T_{vac} = -1$, the tachyon function (16) takes the value $f(T_{vac}) = \frac{1}{2}$ which guarantee the stability of the theory [10].

The equations (12)-(15) can be solved using the following ansatz for the metric

$$ds_{10}^2 = g_{00}(r)dt^2 + g(r)(d\vec{x})^2 + g_{rr}(r)dr^2 + g_S(r)d\Omega_5$$

Moreover one can find the electrically charged three-brane if the following ansatz for the RR field

$$C_{0123} = A(r), F_{0123r} = A'(r) \tag{17}$$

and a constant value for the dilaton field $\Phi = \Phi_0$ is used

$$g_{00} = -H^{-\frac{1}{2}}, g(r) = H^{-\frac{1}{2}}, g_S(r) = H^{\frac{1}{2}}r^2, g_{rr}(r) = H^{\frac{1}{2}}, H = 1 + \frac{e^{\Phi_0}Q}{2r^4} \tag{18}$$

4 Brane-inflation

We consider a D3-brane moving along a geodesic in the background of a type 0 string. The induced metric on the brane (6) using the background solution (18) is

$$d\hat{s}^2 = (-H^{-\frac{1}{2}} + H^{\frac{1}{2}}\dot{r}^2 + H^{\frac{1}{2}}r^2 h_{ij}\dot{\varphi}^i\dot{\varphi}^j)dt^2 + H^{-\frac{1}{2}}(d\vec{x})^2 \tag{19}$$

From eq.(15) the RR field $C = C_{0123}$ using the ansatz (17) becomes

$$C\;' = 2Qg^2 g_s^{-\frac{5}{2}}\sqrt{g_{rr}}f^{-1}(T) \tag{20}$$

where Q is a constant.Using again the solution (18) the RR field can be integrated to give

$$C = e^{-\Phi_0} f^{-1}(T)(1 + \frac{e^{\Phi_0} Q}{2r^4})^{-1} + Q_1 \tag{21}$$

where Q_1 is a constant. The effective density on the brane (10), using eq.(18) and (20) becomes

$$\frac{8\pi}{3}\rho_{eff} = \frac{1}{4}[(f^{-1}(T) + EHe^{\Phi_0})^2 - (1 + \frac{\ell^2 e^{2\Phi_0}}{2}H)]\frac{Q^2 e^{2\Phi_0}}{r^{10}} H^{-\frac{5}{2}} \tag{22}$$

where the constant Q_1 was absorbed in a redefinition of the parameter E. Identifying $g = \alpha^2$ and using $g = H^{-\frac{1}{2}}$ we get from (22)

$$\begin{aligned}\frac{8\pi}{3}\rho_{eff} \quad = \quad & (\frac{2e^{-\Phi_0}}{Q})^{\frac{1}{2}}\Big[\Big(f^{-1}(T) + \frac{Ee^{\Phi_0}}{\alpha^4}\Big)^2 - \Big(1 + \frac{\ell^2 e^{2\Phi_0}}{\alpha^6}(\frac{2e^{-\Phi_0}}{Q})^{\frac{1}{2}} \\ & (1-\alpha^4)^{\frac{1}{2}}\Big)\Big](1-\alpha^4)^{\frac{5}{2}}\end{aligned} \tag{23}$$

From the relation $g = H^{-\frac{1}{2}}$ we find

$$r = (\frac{\alpha^4}{1-\alpha^4})^{\frac{1}{4}}(\frac{Qe^{\Phi_0}}{2})^{\frac{1}{4}} \tag{24}$$

This relation restricts the range of α to $0 \leq \alpha < 1$, while the range of r is $0 \leq r < \infty$. We can calculate the scalar curvature of the four-dimensional universe as

$$R_{brane} = 8\pi(4 + \alpha\partial_\alpha)\rho_{eff} \tag{25}$$

If we use the effective density of (23) it is easy to see that R_{brane} of (25) blows up at $\alpha = 0$. On the contrary if $r \to 0$,then the ds^2_{10} of (1) becomes

$$ds^2_{10} = \frac{r^2}{L}(-dt^2 + (d\vec{x})^2) + \frac{L}{r^2}dr^2 + Ld\Omega_5 \tag{26}$$

with $L = (\frac{e^{\Phi_0}Q}{2})^{\frac{1}{2}}$.This space is a regular $AdS \times S^5$ space.

Therefore the brane develops an initial singularity as it reaches $r = 0$, which is a coordinate singularity and otherwise a regular point of the AdS_5 space. This is another example in Mirage Cosmology [9] where we can understand the initial singularity as the point where the description of our theory breaks down.

If we take $\ell^2 = 0$, set the function $f(T)$ to each minimum value and also taking $\Phi_0 = 0$, the effective density (23) becomes

$$\frac{8\pi}{3}\rho_{eff} = (\frac{2}{Q})^{\frac{1}{2}}\Big((2+\frac{E}{\alpha^4})^2 - 1\Big)(1-\alpha^4)^{\frac{5}{2}} \tag{27}$$

As we can see in the above relation, there is a constant term, coming from the tachyon function $f(T)$. For small α and for some range of the parameters E and Q it gives an inflationary phase to the brane cosmological evolution. In Fig. 1 we have plotted ρ_{eff} as a function of α for $Q = 2$. Note here that E is constrained from (7) as $C + E \geq 0$. In our case using (21) we get $E \geq -2\alpha^4$, therefore E can be as small as we want.

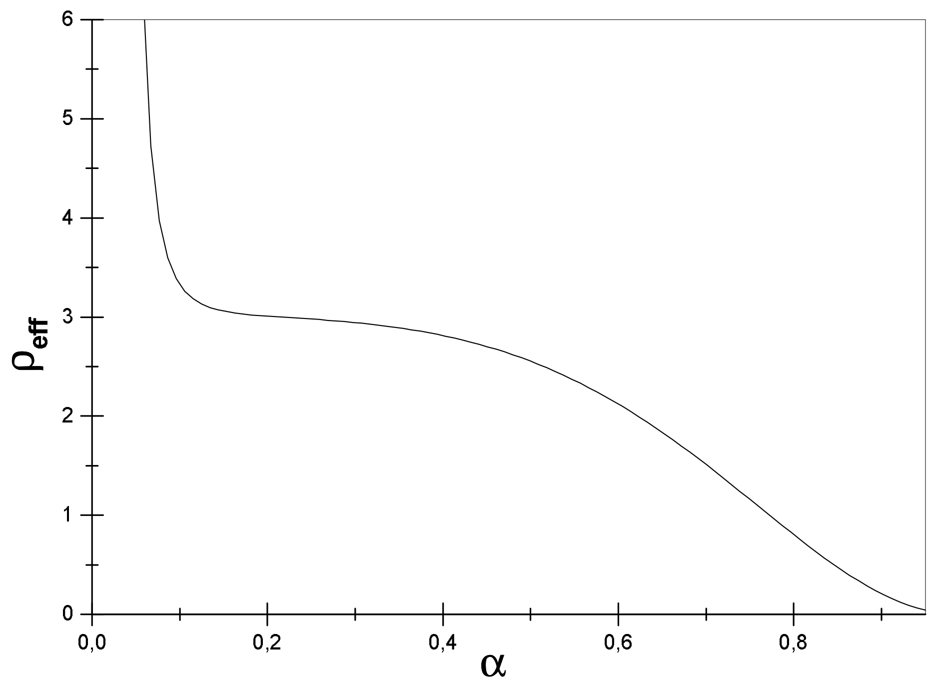

Figure 1: The induced energy density on the brane as a function of the brane scale factor.

As the brane moves away from $r = 0$ to larger values of r, the universe after the inflationary phase enters a radiation dominated epoch because the

term α^{-4} takes over in (27). As the cosmic time η elapses the α^{-8} term dominates and finally when the brane is far away from $r = 0$, the term which is controlled by the angular momentum ℓ^2 gives the main contribution to the effective density. Non zero values of ℓ^2 will give negative values for ρ_{eff}. We expect that at later cosmic times there will be other fields, like gauge fields, which will give a different dynamics to the cosmological evolution and eventually cancel the negative matter density.

5 Discussion

We had followed the movement of a probe brane along a geodesic in the background of type 0 strings. Assuming that the universe is described by a three-dimensional brane, we found that the movement of the brane in the background, induces a cosmological evolution of the universe. As the brane-universe moves along smaller values of the radial coordinate r it contracts while as r becomes larger it expands. An observer on the brane will see, after the initial singularity, the universe to enter an inflationary period and as the scale factor α grows to larger values, the universe to be dominated by radiation. The background we considered is type 0 string. To get an exact solution in an $AdS \times S^5$ background we consider a vacuum where the tachyon T and the dilaton Φ are constant. In this background the tachyon function $f(T)$ which appears as a coupling of the tachyon to the RR field is a constant. In our analysis this constant value gives the inflationary phase of the brane-universe evolution. If we move away from the minimum of $f(T)$, and the dilaton field is not a constant, the equations (12)-(15) cannot be solved exactly. However there are approximate solutions to the above equations in the infrared and the ultraviolet [16]. Work is in progress [17] to develop a realistic phenomenological model for "Mirage Inflation" in which all the astrophysics constraints will be satisfied. We also study the effect of a non constant tachyon and dilaton field to the cosmological evolution of the brane-universe.

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